

## PROBABILITY

### EXAMPLE 1: Finding a Sample Space and Using It to Compute Simple Probabilities

Suppose we roll one fair 6-sided die with one hand and flip a fair 2-sided coin with the other hand.

An example of a simple event that might result from this procedure is getting a 6 on the die paired with heads on the coin.

Another example of a simple event is getting a 5 on the die paired with tails on the coin.

For the complete sample space we must pair every outcome from the roll of the die with every outcome from the coin flip. Let H represent heads and let T represent tails. The complete sample space is {1-H, 1-T, 2-H, 2-T, 3-H, 3-T, 4-H, 4-T, 5-H, 5-T, 6-H, 6-T}

Use this sample space to compute the following:

$$1. P(\text{getting 4 paired with heads}) = \frac{\text{number of outcomes with 4-H}}{\text{size of sample space}} = \frac{1}{12}$$

$$2. P(\text{getting any even number paired with tails}) = \frac{\text{number of even numbers paired with tails}}{\text{size of sample space}} = \frac{3}{12} = \frac{1}{4}$$

Notice that the elements in the sample space that we want to count are 2-T, 4-T and 6-T. These are the three events that are in our numerator.

$$3. P(\text{getting two heads}) = 0$$

This is an impossible event since we are not flipping two coins.

$$4. P(\text{getting a number less than 3 paired with heads}) = \frac{\text{number of numbers less than 3 paired with heads}}{\text{size of sample space}} \\ = \frac{4}{12} = \frac{1}{3}$$

## EXAMPLE 2: Computing Probability from a Contingency Table

In a survey of a recent Statistics class at a commuter college in Florida, students were asked if they owned their own home, rented an apartment or house, or lived with their parents. The table below shows the responses:

	Own Home	Rent	Live with Parents
Male	3	12	5
Female	2	7	9

- How many students were surveyed in this class?

Add up the entries in each cell:  $3 + 12 + 5 + 2 + 7 + 9 = 38$  total students.  
This is the sample size, so  $n = 38$ .

- What is the probability of selecting one student from this class and getting a male who owns his own home?

$$P(\text{male and owns home}) = \frac{\text{number of male homeowners}}{\text{total number in sample}} = \frac{3}{38} \approx 0.0789$$

Round decimals to three significant digits.

- What is the probability of selecting one student from this class and getting a female?

$$P(\text{female}) = \frac{\text{total number of females}}{\text{total number in sample}} = \frac{18}{38} \approx 0.474$$

Round decimals to three significant digits.

- What is the probability of selecting one student from this class and getting a male **or** someone who is renting a house or apartment?

$$\begin{aligned} P(\text{male or renting}) &= \frac{\text{number of males} + \text{number of renters} - \text{number of male renters}}{\text{total number in sample}} \\ &= \frac{20 + 19 - 12}{38} = \frac{27}{38} \approx 0.711 \end{aligned}$$

Round decimals to three significant digits.

The word “**or**” in the question is our clue to use the addition rule.

Notice that when we count the number of events that belong in the numerator we must be sure not to count those male renters twice—we can count them as males or we can count them as renters but we cannot include them in both counts—they are the same guys!

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5. What is the probability of selecting one student from this class and getting someone who does not live with parents?

We will use the concept of complements to answer this question.

$$P(\text{not } A) = 1 - P(A)$$

$$P(\text{not living with parents}) = 1 - P(\text{living with parents})$$

$$= 1 - \frac{\text{number living with parents}}{\text{total number in sample}}$$

$$= 1 - \frac{14}{38} = \frac{38}{38} - \frac{14}{38} = \frac{24}{38} \approx 0.632$$

Round the decimal to three significant digits.

6. What is the probability that one student is selected and that student is a renter given that the student is known to be a male?

$$P(\text{renter/male}) = \frac{\text{number of male renters}}{\text{total number of males}} = \frac{12}{20} = \frac{3}{5} = 0.6$$

Notice that the sample space has changed because we know we have selected a male. The denominator is now the total number of males. The numerator counts only the males who are renters.

7. What is the probability of selecting one student from this class and getting a male or a female?

Since everybody in this sample is either male or female, we are describing a guaranteed or certain event, so the probability is 1.

$$P(\text{male or female}) = P(\text{male}) + P(\text{female}) = \frac{20}{38} + \frac{18}{38} = \frac{38}{38} = 1$$

8. What is the probability of selecting two students from this class (without replacement) and getting two females?

The “**and**” word and the act of selecting two students tells us to use the multiplication rule.

$$P(\text{selecting 2 females}) = P(1^{\text{st}} \text{ selection is female and } 2^{\text{nd}} \text{ selection is female})$$

$$= P(1^{\text{st}} \text{ selection is female}) \cdot P(2^{\text{nd}} \text{ selection is female})$$

$$= \frac{\text{number of females in first selection}}{\text{total number of students}} \cdot \frac{\text{number of females remaining in second selection}}{\text{total number of students remaining in second selection}}$$

$$= \frac{18}{38} \cdot \frac{17}{37} \approx 0.218$$

Notice that for the second selection the number of females decreases by 1 since one female has been removed and the size of the overall sample space decreases by one for the same reason.

### EXAMPLE 3: Computing Probability for at Least One

The probability of an automatic window button working on a five-year-old Ford car is 0.984. A student owns a five-year-old car with four automatic window buttons. What is the probability that at least one of her automatic window buttons is not working?

Let  $W$  represent a window button working on a five-year-old Ford car.

$$P(W) = 0.984$$

Now, we are looking for  $P(\text{at least one out of four window buttons is not working})$ .

The complement of “at least one” is “none”.

$$\begin{aligned} P(\text{at least one out of four buttons is not working}) &= 1 - P(\text{none of the buttons are not working}) \\ &= 1 - P(\text{all of the buttons are working}) \\ &= 1 - P(W)^4 \\ &= 1 - (0.984)^4 \\ &\approx 0.938 \end{aligned}$$

The probability that at least one of her automatic window buttons is not working is 0.938.

### EXAMPLE 4: Counting and Probability

In Florida’s Lotto lottery game, winning the jackpot requires that you select six different numbers from 1 to 53, and the same six numbers must be drawn in the lottery. The winning numbers can be drawn in any order.

- How many different lottery tickets are possible?  
Since the order does not matter (the lottery machine doesn’t know which number you bubbled in first), this counting problem will result in a combination.

$${}_n C_r = \frac{n!}{r! \cdot (n - r)!}$$

There are 53 numbers to select from,  $n = 53$ . We are selecting six numbers. Therefore,  $r = 6$ . Now, we substitute into the formula (or use a calculator) and simplify.

$${}_{53} C_6 = \frac{53!}{6! \cdot (53 - 6)!} = 22,957,480$$

There are 22,957,480 different lottery tickets possible.

- Find the probability of winning the jackpot when one ticket is purchased?  
The size of the sample space is the value that we calculated in part 1 above, 22,957,480.  
There is only one winning combination.

$$\begin{aligned} P(\text{win Florida lottery}) &= \frac{\text{number of winning combinations}}{\text{total number of possible lottery tickets}} \\ &= \frac{1}{22,957,480} \approx 4.36 \times 10^{-8} \end{aligned}$$

The probability of winning the Florida lottery is  $4.36 \times 10^{-8}$ , which would require a lot of luck!