Exponential and Logarithmic Properties

Logarithms

- Logarithms are the inverses of exponents.
- Logarithms are only defined for positive values, X and Y.
- Notation: $\log_b(X)$ is read as "the logarithm base b of X".
- If no base is shown then we assume the base is 10. $\big(\log(X) = \log_{10}(X)\big)$.

Definition of a Logarithm	$\log_b(X) = Y$ if and only if $b^Y = X$
Inverse Properties	$b^{\log_b(X)} = X, \qquad \log_b(b^X) = X$
Identities	$\log_b(b) = 1, \qquad \log_b(1) = 0$
Log Properties	$\log_b(XY) = \log_b(X) + \log_b(Y)$ $\log_b\left(\frac{X}{Y}\right) = \log_b(X) - \log_b(Y)$ $\log_b(X^p) = p\big(\log_b(X)\big)$
One-to-One Properties	If $b^X = b^Y$ then $X = Y$ If $\log_b(X) = \log_b(Y)$ then $X = Y$ If $X = Y$ then $\log_b(X) = \log_b(Y)$

Natural Logarithms

- $\ln(X)$ is known as the natural logarithm of X of base e. $\big(\ln(X) = \ln_e(X)\big)$
- Used to represent $\log_e X$
- Natural logarithms are only defined for positive values, X and Y.

Definition of a Natural Log.	$\ln(X) = Y$ if and only if $e^Y = X$
Inverse Properties	$e^{\ln(X)} = X, \qquad \ln(e^X) = X$
Identities	$\ln(e) = 1, \qquad \ln(1) = 0$
Natural Log Properties	$\ln(XY) = \ln(X) + \ln(Y)$ $\ln\left(\frac{X}{Y}\right) = \ln(X) - \ln(Y)$ $\ln(X^p) = p\big(\ln(X)\big)$
One-to-One Properties	If $b^X = b^Y$ then $X = Y$ If $\ln(X) = \ln(Y)$ then $X = Y$ If $X = Y$ then $\ln(X) = \ln(Y)$