MATH-213: ANALYTIC GEOMETRY AND CALCULUS III

Effective Term
Fall 2026

BOT Approval Date
12/11/2025

Discipline
MATH - Mathematics

Course Number
213

Course Title
Analytic Geometry and Calculus III

Short Title
Analytic Geom/Calc III

Credit Status (CB 04)
D - Credit - Degree applicable

Units

Lecture Units
5.00

Total Units
5.00

Hours

Lecture Contact Hours
80-90

Out of Class Hours Lecture
160-180

Total Contact Hours
80 - 90

Total Out of Class Hours
160 - 180

Total Student Learning Hours
240 - 270

Distance Education
Yes

Distance Education

Distance Education Type
Hybrid Only

What will be done in face-to-face meetings
Face-to-face meeting activities can be varied. The instructor may spend some time reviewing or lecturing on especially challenging topics. Students may work together in small groups to explore multivariable calculus problems and ideas under the supervision of the instructor. A flipped classroom model may be used, and students may work on individual formative assessments in a setting where

they can easily and freely discuss course concepts with the instructor and other students. Summative assessments, such as exams or quizzes, may be occasionally administered.

Students will be required to attend all scheduled face-to-face meetings. Hybrid courses will be scheduled to meet face-to-face at least five times during the semester. Hybrid courses may meet face-to-face up to once a week during a regular semester.

Are you using publisher content?

No

Regular Substantive Interaction

Instructor-to-student, student-to-student, and student-to-content contact shall be distributed in a manner that will ensure regular substantive interaction (RSI) is maintained not only weekly but also for the duration of the course. Instructor interactions should be equivalent in time, quality and consistency to the engagement students would receive in a face-to-face course—for example, through announcements, discussion participation, personalized feedback, video messages, or other instructional communications. Instructors are expected to be responsive to student inquiries within a 24-72 hour time frame excluding holidays. RSI will be maintained through a variety of methods: • Orientation - Students must be oriented to the online and face-to-face portions at the start of the course. • Announcements - Announcements using the course management system must be posted at least weekly to keep students current on course events, due dates, materials, etc. • Discussion Forums - Content-related discussion forums within the designated CMS will include appropriate instructor participation and will be initiated and maintained, with timely feedback provided. Instructors are required to monitor the discussions and related activities to provide guidance and direction. Open-ended discussion boards may be used to further learning and interaction. • MSJC Communication – Instructor will use MSJC-administered communication tools (such as email or the designated CMS communication tools) to regularly communicate with and provide ease of access to students. • Timely Feedback - Timely feedback on student work will be provided by the instructor, which may include comments and/or rubrics, within the Grades area of the designated CMS. • Scheduled Meetings – Hybrid and synchronous courses will meet at regularly scheduled times. ADA Compliance - All course interactions and content must be designed with accessibility in mind and meet ADA requirements.

C-ID (Admin Only)

C-ID MATH 230

Catalog Description

This course covers vectors in two and three dimensions, partial derivatives, multiple integrals, vector fields, Green's theorem, Stokes' theorem, and the divergence theorem.

Requisites

Prerequisite(s)

Prerequisite(s) (must be taken before)
MATH-C2220 (with a grade of C or better).

Codes

TOP Code (CB 03)

1701.00 - Mathematics, General

CIP Code

27.0101 - Mathematics, General.

Repeatability Code

No

Grading Option

Letter grade OR P/NP

Minimum Qualifications

Minimum Qualifications	And/Or
Mathematics (Masters Required)	End

GE Information

Check GE Types Requested
AA/AS G - Math Competency

Cal-GETC Areas

Approved Cal-GETC Area
AREA-2

Learning Objectives

Learning Objectives

Learning Objective
1. Evaluate vector operations.
2. Construct the equations of lines, planes, and quadric surfaces in three-dimensional space.
3. Deduce the limit of a function at a point.
4. Evaluate derivatives of vector-valued functions and partial derivatives of multivariable functions.
5. Construct the equation of a plane tangent to a given surface at a point.
6. Determine differentiability of multivariable functions.
7. Calculate local extrema and test for saddle points of multivariable functions.
8. Solve constraint problems using Lagrange multipliers.
9. Compute the arc length of curves in two- and three-dimensional space.
10. Calculate the divergence and curl of a vector field.
11. Evaluate two and three dimensional integrals.
12. Evaluate integrals using Green's theorem, Stoke's theorem, and the Divergence theorem.
13. Discuss the multicultural origins of calculus, vector analysis, and other fields of mathematics and the diverse set of mathematicians who contributed to these areas.

Learning Outcome Information

Course Learning Outcomes

Learning Outcome
1. Students will apply mulivariable calculus concepts to solve real-world problems, such as problems involving motion, volume, and optimization.
2. Students will interpret and clearly communicate mathematical results graphically, numerically, and symbolically.
3. Students will analyze multivariable and vector-valued functions to identify patterns related to rates of change.

Content

Course Lecture Content

1. Vectors and vector operations in two and three dimensions.
   a. Vector addition, subtraction, and scalar multiplication.
   b. The dot product and its properties.
   c. The cross product and its properties.
   d. The scalar triple product and vector triple product.
   e. Orthogonal projection.
2. Equations of lines and planes.
   a. Vector equations of lines and planes.
   b. Parametric equations of lines and planes.
   c. The rectangular equation of a plane in three-dimensional space.
3. Vector-valued functions.

a. Limits and continuity and their properties.
b. Derivatives and integrals.
c. Velocity and acceleration.
d. Arc-length.
e. Curvature.
f. The unit tangent, normal, and binormal vectors.

4. Real-valued functions of several variables.
   a. Level curves and surfaces.
   b. Limits and continuity and their properties.
5. Differentiability and differentiation of multivariable functions.
   a. Partial derivatives, including higher-order partial derivatives.
   b. The gradient.
   c. The multivariable chain rule.
   d. Directional derivatives.
   e. Local and global extrema.
   f. Saddle points.
   g. Lagrange multipliers.
6. Double and triple integrals.
   a. Finding the limits of integration.
   b. Evaluating the integral.
   c. Integrals in polar, cylindrical, and spherical coordinates.
   d. Applications of multiple integration such as area, volume, center of mass, or moments of inertia.
   e. The change of variables theorem.
7. Vector calculus.
   a. Vector fields including the gradient vector field and conservative fields.
   b. Divergence and curl.
   c. Line and surface integrals including parametrically defined surfaces.
   d. Integrals of real-valued functions over surfaces.
   e. Green's theorem, Stokes' theorem, and the divergence theorem.
8. The multicultural origins of calculus, vector analysis, and other fields of mathematics and the diverse set of mathematicians who contributed to these areas.
   a. Cultures that contributed to the foundations of calculus, vector analysis, or other fields of mathematics, such as the ancient Egyptian, Babylonian, Chinese, Indian, Arabic, or Greek civilizations.
   b. Mathematicians who contributed to calculus, vector analysis, or other fields of mathematics, such as Zeno of Elea, Liu Hui, Bhāskara II, Isaac Newton, or Emily Noether.

Methods of Instruction

Method

Discussion

Integration

Diverse groups of students will discuss how to apply concepts and techniques in such areas of multivariable calculus as vector operations, computing partial derivatives, and evaluating multiple integrals. The instructor will encourage student interaction to discuss the multicultural origins of and the diverse set of mathematicians who contributed to calculus and other fields of mathematics. The instructor will lead discussions of prior positive and negative experiences with mathematics and strategies of how to succeed in mathematics.

DE Adaptations for MOI

Group discussions will be completed using online discussions delivered through the CMS. Synchronous group interactions will take place via accessible video conferencing tools accessed via the CMS that allow for online document sharing. The instructor will ensure that accessibility requirements are met.

Method

Homework

Integration

Homework assignments will allow students to practice applying multivariable calculus concepts and techniques to various problems, including finding the divergence and curl of a vector field, determining the local extrema of a continuous multivariable function, and applying Green's theorem to evaluate a line integral.

DE Adaptations for MOI

Students will access homework assignments via the CMS. They will complete homework problems with paper and pencil, then submit their work through the CMS by uploading a photo or scan of their work, by submitting solutions typed up using a rich text editor or math typesetting tool, or via a mathematics assessment system, while ensuring that accessibility requirements are met. The instructor will review their work and provide feedback through the CMS within one week after the assignment due date.

Method

Lecture

Integration

The instructor will lecture to present multivariable calculus terminology, explain the theory of vectors and higher-dimensional space, and illustrate how to solve problems such as finding the directional derivative of a multivariable function, evaluate a multivariable integral using a change of variables, and determining if a critical point is a local maximum, local minimum, or saddle point. During lecture, the instructor will encourage student feedback and interaction with the instructor.

DE Adaptations for MOI

Lecture content will be communicated through pre-recorded videos, live teleconferencing, and/or text and images delivered through the CMS. The instructor will ensure that accessibility requirements are met.

Method

Observation and Demonstration

Integration

Students will observe the instructor demonstrate techniques to evaluate answers to multivariable calculus problems, such as how to calculate the dot product of two vectors, how to take the partial derivative of a multivariable function, and how to take the curl of a vector field. Also, technology based demonstrations will use software to visualize 3D functions and operations.

DE Adaptations for MOI

Students will observe the demonstrations through live teleconferencing and/or pre-recorded videos shared through the CMS. The instructor will ensure that accessibility requirement are met.

Method

Activity

Integration

Students will work in groups of 3 to 4 people in class to practice multivariable calculus problems, such as finding the equation of a plane that passes through three given points, graphing level curves of quadric surfaces, and using Lagrange multipliers to solve constrained optimization problems.

DE Adaptations for MOI

Group activities will be completed using group discussions and group conversations delivered through the CMS. Synchronous group interactions will take place via accessible video conferencing tools accessed via the CMS that allow for online document sharing. The instructor will ensure that accessibility requirements are met.

Methods of Evaluation

Method

Exams/Tests

Integration

Exams will be used as summative assessments and will be graded on completeness and correctness. Students be evaluated for their conceptual understanding and the correct application of calculating the cross product of two vectors, finding the curvature of a vector-valued function, and computing the tangent plane to a surface at a point.

DE Adaptations for MOE

Students will complete exams with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work, by submitting solutions typed up using a rich text editor or math typesetting tool, or via a mathematics assessment system. Students will receive feedback of their work through the CMS.

Method

Homework

Integration

All homework will be used as formative assessments and will be graded on completeness and accuracy. Students will be graded on their ability to apply mathematics concepts to answer assigned problems on vector-valued functions, multivariable limits, and conservative vector fields.

DE Adaptations for MOE

Students will access homework assignments via the CMS. Students will complete homework assignments with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work, by submitting solutions typed up using a rich text editor or math typesetting tool, or via a mathematics assessment system. The instructor will ensure that accessibility requirements are met. Students will receive feedback of their work through the CMS within one week after the assignment due date.

Method

Quizzes

Integration

Quizzes will be used as formative assessments and will be graded on completeness and correctness. Students will be graded on multivariable calculus topics such as applying the chain rule to differentiate multivariable functions, changing to polar coordinates in a double integral, and applying the divergence theorem to solve a triple integral.

DE Adaptations for MOE

Students will access quizzes through the CMS. Students will complete quizzes with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work, by submitting solutions typed up using a rich text editor or math typesetting tool, or via a mathematics assessment system. The instructor will ensure accessibility requirements are met. Students will receive feedback of their work through the CMS within one week after the assignment due date.

Method

Final Performance

Integration

The final exam may be used as a summative assessment and will be graded on correctness and completeness. Students will be evaluated on course objectives such as their ability to calculate and interpret the gradient of a multivariable function, find the absolute extrema of a continuous function over a bounded domain, and evaluate a triple integral.

DE Adaptations for MOE

Students will access the final exam through the CMS. Students will complete the final exam with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work, by submitting solutions typed up using a rich text editor or math typesetting tool, or via a mathematics assessment system. The instructor will ensure accessibility requirements are met. Students will receive feedback of their work through the CMS within one week after the assignment due date.

Method

Class Work

Integration

Class work will be used as a formative assessment and will be graded on completeness, correctness, and clarity. Students will practice solving multivariable calculus problems such as calculating surface integrals, determining at which points a multivariable function is discontinuous, and modeling and analyzing three-dimensional motion in space.

DE Adaptations for MOE

Students will access class work through the CMS. Students will complete classwork with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work, by submitting solutions typed up using a rich text editor or math typesetting tool, or via a mathematics assessment system. The instructor will ensure accessibility requirements are met. Students will receive feedback of their work through the CMS within one week after the assignment due date.

Method

Oral Presentation

Integration

Students will research and orally present a report on some of the multicultural origins of calculus, vector analysis, and other fields of mathematics or the diverse set of mathematicians who contributed to these areas. Example topics may include calculus advances and techniques by ancient cultures, such as those in Egypt, Babylonia, China, the Middle East, and India. Or students may present a report on an influential mathematician, such as Archimedes, Liu Hui, Hasan Ibn al-Haytham, Bhāskara II, Johannes Kepler, Isaac Newton, René Descartes, Emily Noether, or Maria Gaetana Agnesi, and their contributions to calculus and/or other mathematical fields. The oral presentations will be evaluated via a rubric and/or individual instructor feedback.

DE Adaptations for MOE

Students will access instructions for the assignment through discussion forums in the CMS. Students will post written research reports to the discussion forums within the CMS. Once their own report is posted, they will review and reply to other students' reports. The instructor will ensure accessibility requirements are met. Students will receive feedback of their work through the CMS via a rubric and/or individual instructor feedback within one week after the assignment due date.

Method

Self-Evaluation

Integration

Self-evaluation essays will be used as formative assessments and will be graded on thoughtfulness, completeness of self-reflection, application of critical thinking, and clarity of presentation. Students will recognize their personal growth and plan how to improve in areas they are struggling. Students will reflect on the study skills and habits to be successful in multivariable calculus.

DE Adaptations for MOE

Self-evaluation instructions will be delivered to students via the CMS. Self-evaluation essays will be submitted via the CMS as an assignment. The instructor will ensure that accessbility requirements are met. Students will receive feedback of their work through the CMS within one week after the due date.

Assignments

Assignment Type

Writing

In-class and/or outside of class

In-class

Formative or Summative

Summative

Assignment

Answer all questions below. Be sure to show all your work and justify all your answers. Your work will be graded based on the completeness and correctness of your work and answers.

1. Find the equation of a plane through the points (3, 2, 0), (-2, 0, -1), and (-2, -2, 3).

2. Let f(x, y) = x^2 + 2y^4 + 4xy. Find all critical points of f and classify each critical point as a local maximum, a local minimum, or a saddle point.

3. Use the method of Lagrange multipliers to find the maximum and minimum values of g(x, y) = x^2 + y^2 - 4x - 3y subject to the constraint x^2 + y^2 = 25.

4. Express the double integral of cos(x^2 + y^2) over R = {(x, y) : 16 < x^2 + y^2 < 49, x < 0} in terms of polar coordinates, then evaluate the integral.

5. Let the vector field F = <\sqrt{x} + 4y, 3x + 4y>, and let C be the boundary curve of region R enclosed by y = 3x - x^2 and the x-axis, oriented counterclockwise. Use the flux version of Green's theorem to evaluate the line integral of F·n over the closed curve C, where n is the outward-oriented unit normal vector of the boundary of R.

How assignment will be adapted in online format

This assignment will be delivered to students via the CMS. Students will solve the assignment problems with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work. The instructor will ensure that accessibility requirements are met. Students will receive feedback of their work through the CMS within one week after the due date.

Assignment Type

Writing

In-class and/or outside of class

Outside of class

Formative or Summative

Formative

Assignment

Answer all questions below. Be sure to show all your work and justify all your answers. Your work will be graded based on the completeness and correctness of your work and answers.

Remember that new math concepts can be challenging when first learning them. Do your best, compare your answers to the correct answers, and don't be afraid to make mistakes. Mistakes are a great learning experience! If you find yourself struggling, please reach out to the instructor. Also, you are encouraged to discuss the problems with other students,
though each student must submit their own work.

1. Let a = <-5, -2, -4> and b = <4, 2, -5>. Calculate a × b. [Answer: a × b = <18, -41, -2>]

2. Let f(x, y) = 2x^2 + 5xy^3 - 5y^4. Find #f(x, y). [Answer: #f(x, y) = <4x + 5y^3, 15xy^2 - 20y^3>]

3. Find the directional derivative of the function f(x, y) = ln(x^4 + y^2) at the point (-1, -3) in the direction of the vector <-1, -1>. [Answer: 1/sqrt(2)]

4. Find the volume of the region under the surface z = xy^2 and above the area in the xy-plane bounded by x=y^2 and x+y=12. [Answer: 1509.2]

5. Find the curl of the vector field F = <6zcos(x), 3zsin(x), 4z>. [Answer: curl(F) = <-3sin(x), 6cos(x), 3zcos(x)>]

How assignment will be adapted in online format

This assignment will be delivered to students via the CMS. Students will solve the assignment problems with paper and pencil, then submit their solutions through the CMS by uploading a photo or scan of their work. The instructor will ensure that accessibility requirements are met. Students will receive feedback of their work through the CMS within one week after the due date.

Assignment Type

Reading

In-class and/or outside of class

In-class
Outside of class

Formative or Summative

Formative

Assignment

You will give a short oral presentation (between two and five minutes) on the multicultural origins of calculus or vector analysis and/or the diverse set of mathematicians who contributed to calculus or vector analysis. Choose one of the following:

1. Research and present on an ancient civilization or culture and the techniques they developed to consider calculus questions and/or other mathematical contributions. Examples civilizations you could research include the Egyptian, Babylonian, Chinese, Indian, Arabic,
and Greek civilizations. You may use any appropriate sources for your research, but here are some resources to help you get started:
- "An overview of Egyptian mathematics", by J J O'Connor and E F Robertson [https://mathshistory.st-andrews.ac.uk/HistTopics/Egyptian_mathematics/]
- "Overview of Chinese mathematics", by J J O'Connor and E F Robertson [https://mathshistory.st-andrews.ac.uk/HistTopics/Chinese_overview/]
- "An overview of Babylonian Mathematics", by J J O'Connor and E F Robertson [https://mathshistory.st-andrews.ac.uk/HistTopics/Babylonian_mathematics/]
- "An overview of Indian Mathematics", by J J O'Connor and E F Robertson [https://mathshistory.st-andrews.ac.uk/HistTopics/Indian_mathematics/]

2. Research and present on the life and contributions of one of the diverse set of mathematicians who have contributed to calculus or other mathematical fields. Examples of mathematicians you could research include Archimedes, Eudoxus of Cnidus, Zeno of Elea, Liu Hui, Zu Chongzhi, Hasan Ibn al-Haytham, Bhāskara II, Madhava of Sangamagrama, Emmy Noether, Isaac Newton, Gottfried Wilhelm Leibniz, Alan Turing, and Katherine Johnson. You may use any appropriate sources for your research, but here are some resources to help you get started:
- "Biographies of Women Mathematicians", hosted by Agnes Scott College. [https://mathwomen.agnesscott.org/women/women.htm] A collection of short biographies of influential women in mathematics.
- "MacTutor Index", hosted by the University of St Andrews. [https://mathshistory.st-andrews.ac.uk/] An index of biographies over 3000 mathematicians. Includes lists of mathematicians from different categories, including African mathematicians, ancient Islamic mathematicians, female mathematicians, African-American mathematicians, and mathematicians organized by birth country.

Your presentation will be evaluated using a detailed rubric and/or individual instructor feedback.

How assignment will be adapted in online format

Students will access instructions for the assignment through discussion forums in the CMS. Students will post written reports to the discussion forums within the CMS. Once their own report is posted, they will review and reply to other students' reports. The instructor will ensure accessibility requirements are met. Students will receive feedback of their work through the CMS via a rubric and/or individual instructor feedback within one week after the assignment due date.

Course Materials

Textbooks

Fehribach, Joseph D (2024). Multivariable and Vector Calculus, 2nd Edition. De Gruyter Brill. ISBN: 9783111392387

Stewart, J (2021). Multivariable Calculus, 9th Edition. Cengage. ISBN: 9780357042922

Larson,R. Edwards,B.,H. (2018). Calculus, 11th Edition Cengage. ISBN: 978-1-337-27534-7

Open Educational Resources

Strang, Gilbert and Herman, Edwin. Calculus Volume 3. OpenStax. (https://openstax.org/details/books/calculus-volume-3)

Guichard, David. Calculus: Early Transcendentals. (https://www.whitman.edu/mathematics/calculus_online/)

Class Size Information

Class Size

30

Class Size Category

C - Remedial or advanced math 30

Diversity and Inclusion

Course Design and Materials: Course will incorporate diverse perspectives, authors, and examples that reflect a variety of cultures, identities, and lived experiences. Instructional materials will be reviewed regularly to ensure representation, inclusivity, and alignment with equity-minded practices across disciplines.

Inclusive Learning Environment: Instructors will establish classroom norms and policies that promote respect, civil discourse, and belonging in both online and face-to-face classes. Clear expectations for communication and engagement will support a learning space where all students feel valued and heard.

Accessible Instruction: Course content will be delivered using accessible design. Materials will follow accessibility guidelines within the CMS, including captioned media, readable document formats, and multiple means of engagement to support diverse learning.

Equitable Teaching Practices: Instructors will use transparent and grading criteria, varied assessment methods, and timely feedback to promote fairness and student success. Opportunities for revision, reflection, and multiple demonstrations of learning may be incorporated when appropriate to the discipline.

Culturally Responsive Pedagogies: Instructional strategies will encourage critical thinking about diverse perspectives and systems of power relevant to the course content. Faculty will foster opportunities for students to connect course material to real-world contexts.

Ongoing Reflection and Improvement: Faculty will engage in continuous professional development related to IDEAA principles and reflect on course practices using metacognition to identify opportunities for increased equity, inclusion, and accessibility.

Explain how diversity and inclusion(e.g., gender, culture/race, sexuality, etc.) is infused into the course.

In the objectives and course content, we included the requirement to research and discuss the multicultural origins of and the diverse set of mathematicians who contributed to calculus, vector analysis, and other fields of mathematics. Assignments support this requirement by giving students the opportunity to research, present on, and discuss the mathematical contributions from a wide range of cultures and from mathematicians of different genders, races, and backgrounds.

Describe how assignments, instruction, evaluation, course content, and interactions are contextualized for diverse learners promoting an equitable course.

The the example formative writing assignment demonstrates how instructions can be written to encourage students to approach difficult problems with a growth mindset, to accept mistakes as part of the learning process, and to seek support when needed. In the methods of instruction, we have specified that during discussions and lecture, the instructor will encourage students to share their own experiences in mathematics, discuss strategies for succeeding in mathematics, and seek student feedback of their grasp of course content. The listed methods of evaluation include multiple formative active-learning assessments, allowing students the opportunity to receive and review instructor feedback to correct mistakes/fill gaps in their understanding prior to completing summative assessments. The course materials have been revised to include OERs option to allow students of all socioeconomic backgrounds to fully participate in the course.

Mathematics can be a difficult subject, and many students have developed math anxiety, a fear of making mistakes, and feelings of inadequacy towards math. The above changes are intended to encourage students of all backgrounds and abilities by communicating that mistakes part of the process of learning mathematics, that all students struggle with mathematics to some degree, and that success in mathematics can be achieved by all students through incremental effort and improvement.

Interactions

Explain how the course will incorporate student-to-student interaction.

For both face-to-face and online modalities, group work will give students the opportunity to collaborate on and practice multivariable calculus problems, such as finding the equation of a plane that passes through three given points, graphing level curves of quadric surfaces, and using Lagrange multipliers to solve constrained optimization problems. In both modalities, students will be given

frequent opportunities to discuss how to apply concepts and techniques in such areas of multivariable calculus as vector operations, computing partial derivatives, and evaluating multiple integrals.

Explain how the course will incorporate instructor-to-student interaction.

In both face-to-face and online modalities, instructors will use discussions to interact with students, discussing the multicultural origins of and the diverse set of mathematicians who contributed to calculus, vector analysis, and other fields of mathematics, and they will discuss their prior positive and negative experiences with mathematics and strategies to success in mathematics. Also, instructors will clarify concepts and pose questions about various multivariable calculus concepts, such as multivariable calculus terminology, the theory of vectors and higher-dimensional space, and illustrate how to solve problems such as finding the directional derivative of a multivariable function, evaluate a multivariable integral using a change of variables, and determining if a critical point is a local maximum, a local minimum, or a saddle point. In both face-to-face and online modalities, instructors will encourage student feedback and interaction with the instructor, and the instructor will provide feedback to students on formative and summative assessments.

Explain how the course will incorporate student-to-content interaction.

In both face-to-face and online modalities, students will reference accessible materials such as textbooks, written lecture materials or transcripts of lectures, illustrative images, charts and graphs, or video or audio lectures. In-class exercises and/or at-home assignments will allow students to practice content material such as multivariable limits, conservative vector fields, and modeling and analyzing three-dimensional motion in space.

Explain how the course will incorporate student-to-self interaction (metacognitive).

In both face-to-face and online modalities, students will use self-evaluation essays to reflect on their personal growth and plan how to improve in areas they are struggling. Students will reflect on the study skills and habits to be successful in multivariable calculus. This course emphasizes thinking both logically and flexibly, and it encourages students to consider different interpretations of the same concepts (such as algebraic and geometric interpretations) by engaging in discussions, assessments, and writing activities.

Key: 2109